Since

$$e' = \frac{\sqrt{b^2 - b'^2}}{b}$$

and

$$b' = \sqrt{y^2 + z^2} = \sqrt{b^2 \cos^2 \theta + a^2 \sin^2 \theta},$$

$$e' = \frac{\sqrt{b^2 - b^2 \cos^2 \theta - a^2 \sin^2 \theta}}{b} = \frac{\sqrt{(b^2 - a^2)(1 - \cos^2 \theta)}}{b} = e \sin \theta.$$

But $z/y = \tan i = (a/b) \tan \theta$, or $\tan \theta = (b/a) \tan i$. Therefore,

$$\sin \theta = \frac{b \tan i}{\sqrt{a^2 + b^2 \tan^2 i}} = \frac{b \tan i}{\sqrt{b^2 - b^2e^2 + b^2 \tan^2 i}} = \frac{\tan i}{\sqrt{\sec^2 i - e^2}}.$$

Substituting for $\sin \theta$, we have

$$e' = \frac{e \tan i}{\sqrt{\sec^2 i - e^2}}.$$

Also solved by ROGER A. JOHNSON, ELBERT H. CLARKE, H. L. OLSON, and the PROPOSER.

2674. Proposed by J. O. MAHONEY, Dallas, Texas.

If two sides of a triangle differ by less than a certain length, e, the two opposite angles will differ by less than a certain quantity λ, expressed in degrees, such that $\lambda < 61e/a$ where a expresses, with a possible error e, the length of the apparently equal sides of the triangle.

SOLUTION BY ROGER A. JOHNSON, Hamline University.

This theorem as stated, is not true. Consider, for instance, the triangle

$$a = 1001, \qquad b = 1000, \qquad c = 99.$$

Here,

$$A = 87° \, 44' \, 32'', \qquad B = 86° \, 35' \, 10'', \qquad C = 5° \, 40' \, 18''.$$

Now, considering the nearly equal angles A and B, we have, in fact, $\lambda = 1.157$, whereas by the formula given, we should have $\lambda = .061$ or less. This is an extreme case, but it will be found that in any triangle in which the nearly equal angles are greater than about 50°, the formula does not hold.

As a matter of fact, the correct expression is

$$\lambda = \frac{180}{\pi} \frac{e}{a} \tan A,$$

where A represents the larger of the nearly equal angles. We will not consider the case that either of these two angles exceeds or equals 90°.

If a and b are two sides of a triangle, α, β, the opposite angles, we have

$$\sin \beta = \frac{\sin \alpha}{a} \cdot b;$$

whence

$$\cos \beta d\beta = \frac{\sin \alpha}{a} db,$$

in radians, or

$$d\beta = \frac{180°}{\pi} \cdot \frac{\sin \alpha}{\cos \beta} \frac{db}{a}$$

For our problem, $d\beta = \lambda$, $db = e$, and $(\sin \alpha)/(\cos \beta)$ may be replaced by the greater of the values $\tan \alpha$, $\tan \beta$, yielding the inequality given above.

Also solved by ELIJAH SWIFT and PAUL CAPRON.

2675. Proposed by E. B. ESCOTT, Kansas City, Mo.